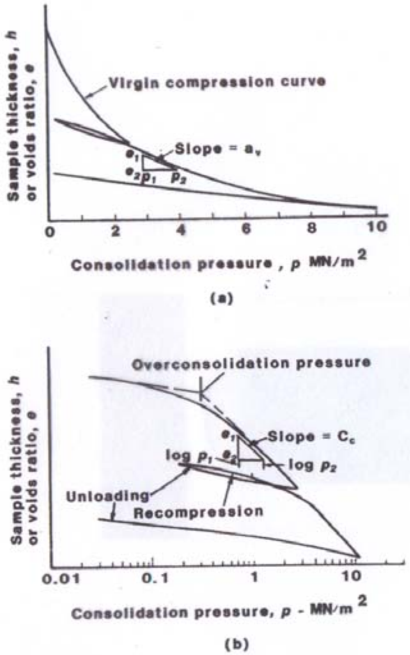

Figure 5.1 Typical plots of compressibility test results

5.1.1 The compressibility parameters

The process of compression on a soil can be usefully illustrated by means of the model soil sample, as illustrated in Figure 5.2. Recognising that compression takes place by a reduction in the volume of voids, with virtually no change in the volume of the solid particles, compressibility was originally defined by the **coefficient of compressibility**, a_v, which is the change in voids ratio per unit increase

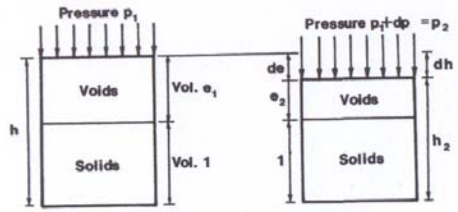

Figure 5.2 Compression of the model soil sample

in pressure. In terms of the model soil sample,

$$a_v = -\frac{de}{dp} = \frac{e_1 - e_2}{p_2 - p_1} \qquad (5.1)$$

and is the slope of the curve shown in Figure 5.1(a) when e is plotted against p. From an engineering viewpoint, it is the proportional change of thickness of a specimen that is of direct concern. For a constant cross-sectional area, this is proportional to the proportional change of volume of a soil, and gives rise to the concept of the **coefficient of volume of compressibility**, m_v, which is much more commonly used:

$$m_v = -\frac{d(\text{volume})}{\text{volume}} \Big/ \frac{1}{dp} = -\frac{dh}{h}\frac{1}{dp} = \frac{h_1 - h_2}{h_1}\frac{1}{(p_2 - p_1)} \qquad (5.2)$$

Referring to the soil sample, m_v can also be expressed in terms of the voids ratio:

$$m_v = \frac{dh}{h_1}\frac{1}{dp} = \frac{de}{1+e_1}\frac{1}{dp} = \frac{e_1 - e_2}{1+e_1}\frac{1}{(p_2 - p_1)} \qquad (5.3)$$

This is the slope of the curve in Figure 5.1(a) when h is plotted against p. From Equations 5.1 and 5.3, the relationship between these two definitions of compressibility is:

$$a_v = m_v(1+e) \qquad (5.4)$$

It can be seen that the slope of the curve in Figure 5.1(a) is not constant. This means that the coefficients a_v and m_v also vary and that a given value applies only to a specific pressure range. However, the curve obtained in figure 5.1(b) when the logarithm of consolidation pressure is used, approximates much more closely to a straight line, at least on the virgin compression curve. This gives rise to two further measures of compressibility, the **compression index**, C_c, and the **modified compression index** or **compression ratio**, C_{ce}, which are the slopes of the virgin compression curves obtained by plotting e or h, respectively, against $\log p$:

$$C_c = -\frac{de}{d(\log p)} = \frac{e_1 - e_2}{\log p_2 - \log p_1} = \frac{e_1 - e_2}{\log(p_2/p_1)} \quad (5.5)$$

$$C_{ce} = -\frac{dh}{h}\Big/ d(\log p) = -\frac{de}{1+e_1}\frac{1}{d(\log p)} = \frac{e_1 - e_2}{1+e_1}\frac{1}{\log(p_2/p_1)} \quad (5.6)$$

Note that, for these evaluations, logarithms are taken to the base 10. From equations 5.5 and 5.6, the relationship between C_c and C_{ce} follows that between a_v and m_v:

$$C_c = C_{ce}(1+e_1) \quad (5.7)$$

Of the two, C_c is much more commonly used. From equations 5.3 and 5.5, it can be related to m_v:

$$\frac{m_v}{C_c} = \frac{e_1 - e_2}{1+e_1} \cdot \frac{1}{(p_2 - p_1)} \Big/ \frac{e_1 - e_2}{\log(p_2/p_1)}$$

giving

$$m_v = \frac{C_c}{1+e_1}\frac{\log(p_2/p_1)}{p_2 - p_1} \quad (5.8)$$

For the compression part of the curve, the terms **recompression index**, C_r, and **modified recompression index**, C_{re}, are used, defined in the same ways as C_c and C_{ce}, respectively.

5.1.2 Settlement calculations using consolidation theory

Returning to the basic definition of the coefficient of volume compressibility, given in equation 5.2:

$$m_v = \frac{dh}{h}\frac{1}{dp}, \quad (5.9)$$

It can be seen that, once m_v is known for a particular pressure range, the compression, dh, of a layer of thickness, h, due to a load increment, dp, can be calculated by simply turning the above equation around:

$$dh = h\,dp\,m_v$$

since dh is normally thought of as the settlement, ρ, and dh is the applied pressure increase, σ, this becomes:

$$\rho = H\sigma m_v \quad (5.10)$$

where specimen thickness, h, is now replaced by thickness, H, of the compressible stratum. The average value of σ across a compressible layer, due to some applied loading, is usually calculated using elasticity theory. Although not strictly valid for soils, it gives sufficiently accurate values. Settlement is then obtained using consolidation theory by way of Equation 5.10.

Where values of C_c are obtained, m_v values may be calculated from Equation 5.8, using the appropriate values of consolidation pressure and voids ratio. Alternatively, Equations 5.8 and 5.10 may be combined and settlement calculated directly from C_c values:

$$\rho = H\sigma m_v = H(p_2 - p_1)\frac{C_c}{1+e}\frac{\log(p_2/p_1)}{(p_2 - p_1)}$$

giving

$$\rho = HC_c\frac{\log(p_2/p_1)}{1+e}$$

5.1.3 Settlement calculations using elasticity theory

An alternative approach is to calculate displacements (settlements) directly using elasticity theory, thus reducing the two separate stages in the settlement calculation to one, and obviating the need to calculate average values of consolidation pressure across soil layers. Numerous solutions, for both stresses and displacements, have been produced, many of which have been presented by Poulos and Davis (1974).

The problem with using elastic solutions to calculate settlements is that it requires the evaluation of Young's modulus, E, and Poisson's ratio, ν, neither of which are measured, or are strictly meaningful, for soil consolidation problems. Considering Equation 5.9, since the ratio dh/h can be thought of as a strain, m_v is strain/stress, with units 1/stress; typically m^2/kN or m^2/MN. Thus, it is by definition akin to the reciprocal of Young's modulus, E, and whereas E can be envisaged simplistically as the stress required to double the length of an object, m_v can be envisaged as an area of soil which, if subjected to a unit load, will just disappear! Of course, such absurdities do not occur in reality because the relationships are not valid for these extremes. Additionally, the relationship between E and m_v is not a simple reciprocal one because E is defined for a specimen with unrestrained sides whereas m_v is defined for a specimen which is laterally constrained. The relationship between E and m_v therefore depends on the value of Poisson's ratio, thus:

$$m_v = \frac{1}{E}\frac{(1+\nu)(1-2\nu)}{(1-\nu)}$$

This relationship can then be used when calculating settlements using elastic theory. When used in this context, E is not strictly an elastic constant, but it does represent the response of the soil to a single loading applied over a long period. To emphasise the point, the term 'deformation modulus' is sometimes used for E defined in this way. Thus, elastic theory can be used to calculate consolidation settlements, even though these are not elastic (i.e. recoverable). The main problem lies in obtaining a value of Poisson's ratio that properly represents the consolidation behaviour of soils. Poisson's ratio is not measured in standard soil testing and, indeed, it is virtually impossible to obtain realistic measurements. However, it has been pointed out by Skempton and Bjerrum (1957) that very little lateral strain occurs during the consolidation of clays so that, effectively, Poisson's ratio is zero, and

$$E = \frac{1}{m_v} = M$$

where M is the deformation modulus or constrained modulus.

Another reason for choosing a zero value is that calculated settlements based on elastic solutions then become identical with those based on consolidation theory, which has been shown over the years to give reasonable predictions provided that suitable corrections are made for the pore pressure response of the soil (Skempton and Bjerrum 1957).

5.1.4 Typical values and correlations of compressibility coefficients

Typical values of the coefficient of volume compressibility, m_v are indicated in Table 5.1, along with descriptive terms for the various ranges of compressibility. Although m_v is the most suitable, and most popular, of the compressibility coefficients for the direct calculation of settlements, its variability with confining pressure makes it less useful when quoting typical compressibilities or when correlating compressibility with some other property. For this reason, the

Table 5.1 TYPICAL VALUES OF THE COEFFICIENT OF VOLUME COMPRESSIBILITY AND DESCRIPTIVE TERMS USED (AFTER CARTER 1983)

Type of clay	*Descriptive term*	*Coefficient of volume compressibility, m_v* (1 m²/MN = 0.1 cm²/kg)	
		(m²/MN)	(ft²/ton)
Heavy over-consolidated boulder clays, stiff weathered rocks (e.g. weathered mudstone) and hard clays	Very low compressibility	<0.05	<0.005
Boulder clays, marls, very stiff tropical red clays	Low compressibility	0.05–0.1	0.005–0.01
Firm clays, glacial outwash clays, lake deposits, weathered marls, firm boulder clays, normally consolidated clays at depth and firm tropical red clays	Medium compressibility	0.1–0.3	0.01–0.03
Normally consolidated alluvial clays such as estuarine and delta deposits, and sensitive clays	High compressibility	0.3–1.5	0.03–0.15
Highly organic alluvial clays and peats	Very high compressibility	>1.5	>0.15

Table 5.2 TYPICAL VALUES OF COMPRESSIBILITY INDEX, C_c (AFTER HOLTZ AND KOVACS 1981)

Soil	C_c
Normally consolidated medium sensitive clays	0.2 to 0.5
Chicago silty clay (CL)	0.15 to 0.3
Boston blue clay (CL)	0.3 to 0.5
Vicksburg Buckshot clay (CH)	0.5 to 0.6
Swedish medium sensitive clays (CL–CH)	1 to 3
Canadian Leda clays (CL–CH)	1 to 4
Mexico City clay (MH)	7 to 10
Organic clays (OH)	4 and up
Peats (Pt)	10 to 15
Organic silt and clayey silts (ML–MH)	1.5 to 4.0
San Francisco Bay Mud (CL)	0.4 to 1.2
San Francisco Old Bay clays (CH)	0.7 to 0.9
Bangkok clay (CH)	0.4

compression index, C_c, is usually preferred. Typical value of compression index are given in Table 5.2.

Skempton (1944) proposed the following relationship between compression index and liquid limit (LL) for normally-consolidated

Table 5.3 SOME PUBLISHED CORRELATIONS FOR COMPRESSION INDICES (AFTER AZOUZ ET AL. 1976)

Equation	*Regions of applicability*
$C_c = 0.007\ (LL - 7)$	Remoulded clays
$C_{ce} = 0.208e_0 + 0.0083$	Chicago clays
$C_c = 17.66 \times 10^{-5} w_n^2 + 5.93 \times 10^{-3} w_n - 1.35 \times 10^{-1}$	Chicago clays
$C_c = 1.15(e_0 - 0.35)$	All clays
$C_c = 0.30(e_0 - 0.27)$	Inorganic, cohesive soil; silt, some clay; silty clay; clay
$C_c = 1.15 \times 10^{-2} w_n$	Organic soils-meadow mats, peats, and organic silt and clay
$C_c = 0.75(e_0 - 0.50)$	Soils of very low plasticity
$C_{ce} = 0.156e_0 + 0.0107$	All clays
$C_c = 0.01w_n$	Chicago clays

As summarised by Azzouz, Krizek, and Corotis (1976).
Note: w_n = natural water content.

clays:

$$C_c = 0.007(LL\text{-}10).$$

Terzaghi and Peck (1967) proposed a similar relationship, based on research with clays of low and medium sensitivity:

$$C_c = 0.009(LL\text{-}10).$$

This relationship has a reliability range of ±30% and is valid for inorganic clays of sensitivity up to 4 (see Chapter 6) and liquid limit up to 100. Based on the work of Skempton and Northey (1952) and Roscoe *et al.* (1958), Wroth and Wood (1978) used critical state soil mechanics considerations to deduce a relationship between compression index and plasticity index (PI) for remoulded clays:

$$C_c = \tfrac{1}{2} PI \cdot G_s$$

where G_s is the specific gravity of the soil solids. Table 5.3 produced by Azzouz *et al.* (1976) gives a summary of a number of published correlations.

The recompression index, C_r, is defined in the same way as C_c except that it applies to the unloading phase of the consolidation test. Typical values of C_r range from 0.015 to 0.35 (Roscoe *et al.* 1958) and are often assumed to be 5–10% of C_c.

5.1.5 Settlement corrections

If the results of oedometer tests are used directly to calculate settlements, the values obtained tend to over-estimate the settlements

that actually occur, particularly with overconsolidated clays. An exception to this is in the case of very sensitive clays, where predicted settlements may slightly under-estimate actual values. The reason for this is that the pore pressure response of clays in the field differs from that of confined laboratory specimens. This has been discussed by Skempton and Bjerrum (1957), who show that the ratio of actual settlement to calculated settlement depends on both the response of the pore water pressures to applied loads and the geometry of each problem. The response of the pore water pressures to loading can be measured in the triaxial test and is expressed in terms of Skempton's (1954) pore pressure parameters, A and B. For saturated clays, actual settlement, ρ_{field}, is given by:

$$\rho_{field} = \mu \cdot \rho$$

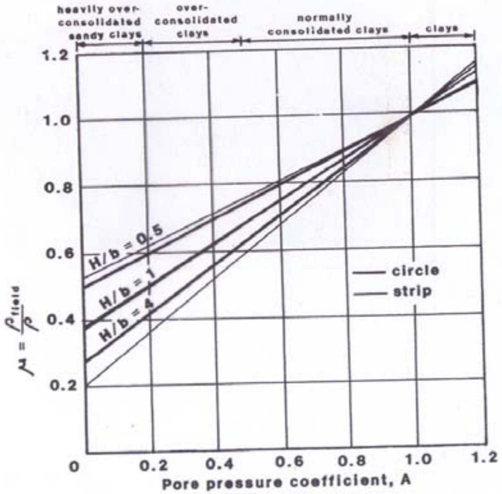

Figure 5.3 Typical values of the factor μ for a foundation width b on a compressible layer of thickness h (after Skempton, 1954)

for clays are given in Table 5.6, and an approximate correlation with liquid limit is shown in Figure 5.4.

5.3 SECONDARY COMPRESSION

Secondary compression is a volume change under load that takes place at constant effective stress; that is, after the excess pore water pressure has dissipated. It is thought to result from compression of the constituent soil particles at a microscopic or molecular scale and is particularly significant in organic soils. Coefficients of secondary compression may be defined in a way that is analogous to the definitions of compression index and modified compression index, except that the indices are related to time instead of pressure. Thus, the secondary compression index, C_α is:

$$C_\alpha = \frac{de}{d(\log t)} \qquad (5.11)$$

where de is the change in voids ratio over a time interval, dt, from time t_1 to time t_2: see Figure 5.5. Similarly, the modified secondary compression index, $C_{\alpha\varepsilon}$ is:

$$C_{\alpha\varepsilon} = \frac{dh/h}{d(\log t)} = \frac{C_\alpha}{1+e_p} \qquad (5.12)$$

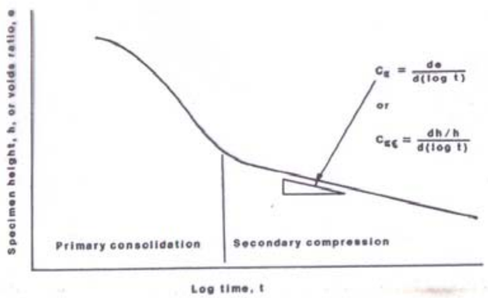

Figure 5.5 Plotting and calculation of secondary compression

where e_p is the voids ratio at the start of the linear portion of the e-$\log p$ (or $dh-\log p$) curve. The modified secondary compression index is sometimes also referred to as the secondary compression ratio or the rate of secondary compression.

Calculations of secondary compression are obtained by rearranging Equation 5.12: specimen compression dh becomes secondary settlment, ρ_c; specimen thickness, h, becomes layer thickness, H; and the time is taken over a specific interval, from t_1 to t_2:

$$\rho_c = C_{\alpha\varepsilon} H \log(t_2/t_1)$$

or

$$\rho_c = \frac{C_\alpha}{1+e} H \log(t_2/t_1)$$

For the purpose of secondary settlement calculations, secondary settlement is assumed to start when primary settlement is substantially complete. Thus, if primary settlements were substantially complete in 12 years, the value of t_1 would be 12. The value of t_2 depends on the assumed lifespan of the structure under consideration.

Values of C_α or $C_{\alpha\varepsilon}$ are obtained from $e-\log p$ or $dh-\log p$ plots, as indicated in Figure 5.5. C_α is usually assumed to be related to C_c, with values of C_α/C_c typically in the range 0.025–0.006 for inorganic soils and 0.035–0.085 for organic soils. Some typical values are given in Table 5.7. Mesri (1973) obtained a relationship between $C_{\alpha\varepsilon}$ and natural moisture content, given in Figure 5.6.

Table 5.7

Soil	C_α/C_c
Organic silts	0.035–0.06
Amorphous and fibrous peat	0.035–0.085
Canadian muskeg	0.09–0.10
Leda clay (Canada)	0.03–0.06
Post-glacial Swedish clay	0.05–0.07
Soft blue clay (Victoria, B.C.)	0.026
Organic clays and silts	0.04–0.06
Sensitive clay, Portland, ME	0.025–0.055
San Francisco Bay Mud	0.04–0.06
New Liskeard (Canada) varved clay	0.03–0.06
Mexico City clay	0.03–0.035
Hudson River silt	0.03–0.06
New Haven organic clay silt	0.04–0.075

* Modified after Mesri and Goldlewski (1977).

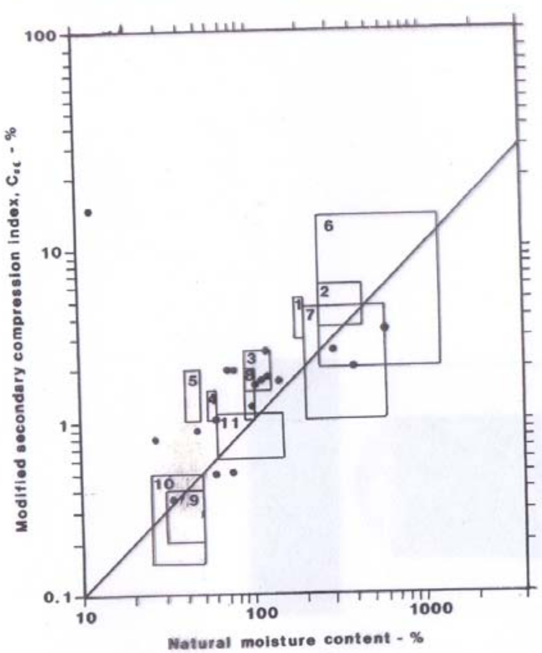

Figure 5.6 Correlation between modified secondary compression index and natural moisture content (after Mesri, 1973)

5.4 SETTLEMENT OF SANDS AND GRAVELS

5.4.1 Probes and standard penetration tests

As mentioned in the introductory remarks to this chapter, the near-impossibility of obtaining and testing undisturbed samples of granular soils means that consolidation testing is not possible. Instead, settlements are usually estimated from insitu test results, most commonly using the standard penetration test, although the use of probes, in the form of static or dynamic cones, has become more

widespread in recent years (ESOPT, 1982; INSITU, 1986; ISOPT 1988). A useful review of the interpretation of some penetration tests for sands is given by Robertson and Campanella (1985).

The most commonly-used correlations for settlement estimates in sands, based on SPT results, are those established by Terzaghi and Peck (1967), shown in Figure 5.7. Terzaghi and Peck point out that the correlations show wide scatter and should not be regarded as anything more than a rough-and-ready guide. Considering the practical problems of obtaining meaningful SPT results, especially in sands below the water table, and the disagreements over various corrections to be applied to the results, the correlations are of dubious value in many cases. Yet settlement estimates are of crucial importance for the determination of allowable foundation pressures on granular soils, whose high ultimate bearing capacity means that

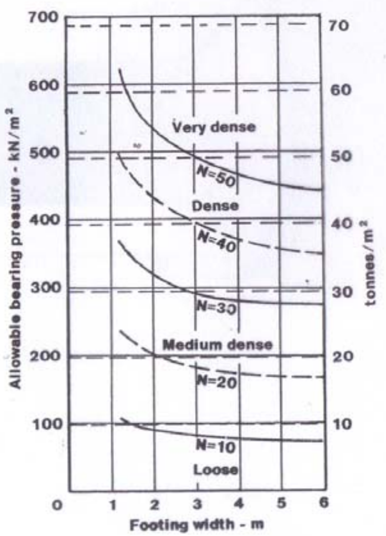

Figure 5.7 Chart for estimating allowable bearing pressures on sands using standard penetration test results, based on 25mm settlement. Continuous lines are based on the original chart by Terzaghi and Peck (1967); broken lines are interpolations